Learning and Predictability via Technical Analysis: Evidence from Bitcoin and Stocks with Hard-to-Value Fundamentals

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Learning and Predictability via Technical Analysis: Evidence from Bitcoin and Stocks with Hard-to-Value Fundamentals

What predicts returns on assets with "hard-to-value" fundamentals, such as Bitcoin and stocks in new industries? We propose an equilibrium model that shows how rational learning can generate return predictability through technical analysis. We document that ratios of prices to their moving averages forecast daily Bitcoin returns in- and out-of-sample. Trading strategies based on these ratios generate an economically significant alpha and Sharpe ratio gains relative to a buy-and-hold position. Similar results hold for small-cap, young-firm, and low-analyst-coverage stocks as well as NASDAQ stocks during the dotcom era.

JEL classification: G11, G12, G14
Keywords: Bitcoin, cryptocurrency, technical analysis, learning, return predictability

Cryptocurrencies' fundamental source of intrinsic value remains unclear. Market observers disagree about their ability to serve as a currency and their currency status faces significant regulatory risk. Moreover, unlike cash flows from more typical financial assets such as stocks and bonds, cryptocurrencies' fundamentals have few, if any, publicly available predictive signals, such as analyst coverage and accounting statements. We refer to fundamentals with these characteristics of uncertainty, opacity, disagreement, and lack of predictive information as "hard-to-value". In this paper, we theoretically and empirically examine the asset-pricing implications of having such fundamentals. While cryptocurrencies are an ideal setting to investigate this asset-pricing property, this property is more general as the fundamentals of most assets are hard-to-value to varying degrees. For example, fundamentals of young small-cap stocks in new industries are harder to value than those of large-cap stocks in established industries.

We propose a continuous-time equilibrium model in which two rational and risk-averse investors costlessly trade a risky asset with hard-to-value fundamentals. This asset produces a stream of benefits called a "convenience yield" that grows at an unobserved and stochastically evolving rate. Investors have different priors and, aside from the convenience yield itself, observe no other signal about the yield's latent growth rate. The risky asset can be interpreted as a cryptocurrency where the convenience yield represents the flow of benefits from usage as a medium of exchange or another asset such as a stock whose dividends or earnings are hard-to-value. In the process of Bayesian learning, investors update their beliefs about the growth rate in the direction of shocks to the convenience yield. However, the initial value of the growth rate is uncertain and these shocks are only imperfectly correlated with unobservable shocks to this rate, causing investors to only gradually move away from their priors when updating beliefs—and consequently valuations—resulting in price drift. Specifically, returns are predictable by ratios of prices to their moving averages (MAs), which summarize the beliefs of investors about the expected convenience yield growth rate. Moreover, because of this predictability, it is optimal for investors to use the price-to-MA ratios in trading. As far as we know, this is the first fully rational general equilibrium model with endogenous use of technical analysis.[1]

[1] Treynor and Ferguson (1985), Brown and Jennings (1989), Cespa and Vives (2012), Brock et al. (1992), Hong and Stein (1999), Lo et al. (2000), Chiarella et al. (2006), Edmans et al. (2015), and Han et al. (2016), among others, show that past prices can predict returns and trading based on technical indicators,

Our model contributes to several strands of literature. It provides a fully rational and endogenous justification for technical analysis and time-series return predictability by past prices, which most practitioners and prior studies justify with irrational forces, such as sentiment, overconfidence, or under-reaction (e.g., Barberis et al. ,1998; Daniel et al., 1998; Hong and Stein, 1999).[2] Moreover, prior models with technical traders assume that a subset of investors use exogenously given technical trading rules (e.g., Hong and Stein, 1999; Han et al., 2016). Our model also proposes a new mechanism relative to the few rational models that generate return predictability by past prices. For example, in prior rational expectations equilibrium models with learning, price drift can arise, but only given higher-order disagreement between traders (e.g., Banerjee et al., 2009). Without this disagreement, agents infer each other's private signals immediately via the price, precluding a gradual drift toward the fundamental value. In our model, drift does not require disagreement, only Bayesian learning and the hard-to-value property of fundamentals. Johnson (2002) also generates price drift for stocks with time-varying dividend growth rates, which are analogous to our convenience yield. However, Johnson does not consider general equilibrium effects, learning, or endogenous technical trading rules.

In the empirical portion of the paper, we investigate whether the predictability of returns by price-to-MA ratios holds for Bitcoin and several equity portfolios with plausibly hard-to-value fundamentals. We find that daily Bitcoin returns are predictable in- and out-of-sample by ratios of prices to their 1- to 20-week MAs. Consistent with our model, this predictability strengthens when uncertainty decreases as investors learn about the dynamics of the latent growth of the convenience yield. Indeed, we find a negative interaction between the price-to-MA ratio and conditional return variance, a proxy for uncertainty, in return-forecasting regressions. To assess the economic significance of this Bitcoin-return predictability to investors, we form a trading strategy that goes long Bitcoin when the price is above the MA, and long cash otherwise. We find that these trading strategies significantly outperform the buy-and-hold benchmark, producing large alphas and increasing Sharpe ratios by 0.2 to 0.6. These results are similar across both halves of the sample. The MA

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especially the moving averages of prices, can be profitable in the stock market. Schwager (1989) and Lo and Hasanhodzic (2009) further provide insightful comments about the effectiveness of technical strategies from top practitioners.

[2] Perhaps the most widely used investments textbook, Bodie et al. (2014), places technical analysis in a chapter on "behavioral finance".

strategies also outperform the buy-and-hold benchmark when applied to two other cryptocurrencies, Ripple and Ethereum, Bitcoin's two largest competitors. Indeed, we find a negative interaction between the price-to-MA ratio and conditional return variance, a proxy for uncertainty, in return-forecasting regressions. To assess the economic significance of this Bitcoin-return predictability to investors, we form a trading strategy that goes long Bitcoin when the price is above the MA, and long cash otherwise. We find that these trading strategies significantly outperform the buy-and-hold benchmark, producing large alphas and increasing Sharpe ratios by 0.2 to 0.6. These results are similar across both halves of the sample. The MA strategies also outperform the buy-and-hold benchmark when applied to two other cryptocurrencies, Ripple and Ethereum, Bitcoin's two largest competitors.

Next, we evaluate whether returns on the NASDAQ portfolio are predictable by price-to-MA ratios during five- and ten-year windows (1998–2002 and 1996–2005, respectively) that includes the dot.com boom-and-bust of the early 2000's. In this period, many emerging technologies associated with the Internet introduced fundamentals that, at the time, were considered difficult to value. We show that our MA trading strategies applied to the NASDAQ generate significant alpha and Sharpe ratio gains (of 0.2 to 0.5) relative to the buy-and-hold benchmark in this time period. Moreover, the gains of the MA strategies steadily decline in the years following this period as fundamentals presumably became easier to value. We also apply our MA strategies to portfolios formed on widely used proxies for information availability: size, age, and analyst coverage. Consistent with our model, we find that over the 1963 to 2018 time period, the price-to-MA ratios positively and significantly forecast returns on small-cap and young-stock portfolios, with negative and insignificant predictability for large-cap and old-stock portfolios. Moreover, in the 1985 to 2018 time-period during which analyst forecasts are available, we find that return predictability by the price-to-MA ratios decreases with both size and analyst coverage.

Our model also suggests that trading arises from differences in the MAs across investors. Consistent with this implication, we show that proxies for disagreement across MA horizons and total turnover implied by the various MA strategies are significantly and positively associated with Bitcoin trading volume.

Overall, consistent with our model, the results in this paper demonstrate that for Bitcoin and stocks with hard-to-value fundamentals, price drift exists and price-to-moving average ratios predict returns.

The rest of the paper is organized as follows. Section II discusses the model and related literature. Section III describes the data. Section IV reports empirical results, and Section V concludes.

2| THE MODEL AND RELATTED LITERAURE

2.1 The Model

In this section, we present a rational equilibrium asset-pricing model that examines the implications of having hard-to-value fundamentals. In the model, investors continuously trade two assets for no cost: a risky asset called "Bitcoin" with one unit of net supply and one risk-free asset with zero net supply.[3]

Assumption 1. *Each unit of Bitcoin provides an observable stream of convenience yield* δ_t*, that grows according to:*[4]

$$\frac{d\delta_t}{\delta_t} = X_t dt + \sigma_\delta dZ_{1t}, \tag{1}$$

where the drift, X_t, is unobservable, and evolves according to:

$$dX_t = \lambda(\bar{X} - X_t)dt + \rho\sigma_X dZ_{1t} + \sqrt{1-\rho^2}\sigma_X dZ_{2t}, \tag{2}$$

where $\sigma_\delta > 0$, $\lambda > 0$, $\bar{X} > 0$, $\sigma_X > 0$, and $\rho \in [-1,1]$ are all known constants and (Z_{1t}, Z_{2t}) is a two-dimensional standard Brownian motion.

While Bitcoin does not provide any cash flows, we assume it must offer some flow of benefits, which we call "convenience yield", δ_t, to its owners, although there is significant and time-varying uncertainty about how these benefits will evolve. For example, holding Bitcoin can facilitate transactions (particularly illicit ones), hedge hyper-inflation risk caused by political turmoil, and serves as a store of value. As a result, investors buy it trading off convenience yield and risks. For other financial assets like stocks and bonds, the

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[3] Obviously, in the real world, there are other risky assets in the economy. As a result, our model will not accurately explain variation in the risk-free rate. In a single-risky-asset general equilibrium model, the risk-free rate will necessarily have non-trivial volatility. However, adding other assets would not change our model's main point that moving average rules can be optimal for trading Bitcoin due to learning. More generally, it is possible to add features to correct counterfactual predictions about the risk-free rate, but only at the expense of parsimony.

[4] One of the features of Bitcoin that are different from traditional assets is that the supply is time varying, because it depends on how much Bitcoin has been mined. However, incorporating this into the model would make the analysis more complicated without changing the main mechanism behind our results.

convenience yield can be interpreted as a dividend, earnings, or interest paid to their owners. The state variable X_t is a catch-all for whatever state variable affects the convenience yield of an asset. For example, in the case of Bitcoin, the state variable may capture uncertain regulatory risks, the likelihood of hyper-inflation in some countries, the popularity of competing cryptocurrencies, and the related technology (e.g., block-chain update speed) advancement. In the case of stocks, the X_t can represent the aggregate of all variables that impact mean dividend growth.

On the investors, we make two assumptions:

Assumption 2 *There are two types of investors who differ by their priors about the state variable* X_t *and possibly initial endowment of Bitcoin.*[5] *Type* i *investor is endowed with* $\eta_i \in (0,1)$ *units of Bitcoin with* $\eta_1 + \eta_2 = 1$ *and has a prior that* X_0 *is normally distributed with mean* $M^i(0)$ *and variance* $V^i(0)$, $i = 1,2$.

Assumption 3 *All investors have log preferences over the convenience yield provided by Bitcoin with discount rate* β *until time* T*. Specifically, the investor's expected utility is*

$$E \int_0^T e^{-\beta t} \log C_t^i dt,$$

where C_t^i denotes the convenience yield received by a Type i investor from owning Bitcoin.

Intuitively, the heterogeneous priors across agents captures the fact that investors have significantly different expectations about how Bitcoin's fundamentals will grow in the future. Assuming log preferences allows for relatively simple and transparent functional forms without altering what would be our main prediction in the case of higher risk aversion.

Denote by F_t^i the filtration at time t generated by the Bitcoin price process $\{B_s\}$ and the prior $(M^i(0), V^i(0))$ for all $s \leq t$ and each investor $i = 1, 2$. Because agents observe only one shock, F_t^i turns out to be equivalent to the filtration based on the convenience yield process $\{\delta_s\}$ in lieu of $\{B_s\}$. Further let $M_t^i \equiv E[X_t | F_t^i]$ be the conditional expectation of X_t given F_t^i. While many risky assets have uncertain and

[5]Since investors can continuously observe δ_t, they can directly calculate the volatility σ_δ and therefore there is no disagreement about the volatility.

stochastically evolving fundamental growth rates, the definition of the filtration and M_t captures the essence of the “hard-to-value" property. That is, Bitcoin investors must filter out the state variable from, at most, the history of convenience yields, and they have initial disagreement about what the convenience yield growth rate is. In contrast, assets with less hard-to-value fundamentals, such as large-cap stocks in established industries, have other signals that help forecast fundamentals, such as thoroughly vetted accounting statements, established correlations with macroeconomic conditions, and analyst coverage.

Proposition 1 *In equilibrium in the economy defined by Assumptions 1–3:*

$$\frac{dB_t}{B_t} = \left((\beta + M_t^i) - \frac{\delta_t}{B_t}\right) dt + \sigma_\delta d\hat{Z}_{1t}^i, \tag{3}$$

the fraction of wealth invested in the Bitcoin by Investor 1 is

$$1 + \frac{\alpha_t}{1+\alpha_t}\frac{M_t^1 - M_t^2}{\sigma_\delta^2}, \tag{4}$$

and by Investor 2 is

$$1 - \frac{1}{1+\alpha_t}\frac{M_t^1 - M_t^2}{\sigma_\delta^2}, \tag{5}$$

where

$$M_t^i = h^i(t) + f^i(0,t)\log\frac{B_t}{B_0} + (f^i(t,t) - f^i(0,t))\left(\log B_t - \frac{\int_0^t g^i(u,t)\log B_u du}{\int_0^t g^i(u,t) du}\right) \tag{6}$$

is the ith investor’s conditional expectation, $E[X_t|F_t^i]$, $h^i(.)$, $f^i(.,.)$, and $g^i(.,.)$ are as defined in the Appendix for $i = 1,2$, and α_t is as defined in Appendix (8), denoting the ratio of the marginal utility of type 1 investor to that of type 2 investor. In addition, if

$$V^i(0) \leq \rho\sigma_X\sigma_\delta + \frac{\sigma_X^2}{\lambda}, \tag{7}$$

then

$$g^i(u,t) > 0, f^i(t,t) - f^i(0,t) > 0, \forall u > 0, t > 0. \tag{8}$$

Proof. See Appendix.

2.2| Discussion and Related Literature

There are two important implications of Proposition 1. First, under Condition (7), the cum-yield Bitcoin return is positively predictable by “ratios" of prices to their moving averages. Intuitively, because of learning, investors only gradually update their priors in the direction of shocks to the convenience yield, resulting in a price drift. If agents had access to other predictive signals, as they would with assets that are not “hard to value", they would incorporate these signals and the predictability of prices by moving averages would no longer be guaranteed.[6] Second, because of the predictive power of the price-to-MAs ratios, every investor’s trading strategy depends on the moving averages.[7] The proof of Proposition 1 shows that learning drives the entire return predictability in a way that is unaffected by the existence of multiple traders, which serves only to create trading. As far as we know, this is the first equilibrium model that justifies the use of moving averages of prices in guiding trading.

Existing models generate price drift via different mechanisms, most of which are based on behavioral biases, such as under- or over-reaction. On the rational side, under certain circumstances, rational expectations equilibrium (REE) models with learning, beginning with the seminal work of Grossman (1976) and Hellwig (1980), also predict price drift. While agents in our model observe a common signal, these models feature agents who receive private signals about fundamental value, and they infer each other’s’ signals from the price. As discussed in Banerjee et al. (2009), price drift in these models requires higher order disagreement to slow down the rate at which agents incorporate each others’ signals in their private valuations. In contrast, agents in our model observe a common signal, and price drift does not depend on multiple traders. Cochrane et al. (2008) also shows that price drift can arise under certain conditions when multiple risky assets exist.

On the irrational side, many studies try to explain price drift, sometimes called time-series (TS)

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[6] One can easily extend this model to include an additional signal about the fundamental. In this extension, the traders would still use MAs as a signal to trade. The less precise the additional fundamental signal, the more weight traders would place on the price-to-MAs. Thus, the predictability of returns by the price-to-moving-average ratios should be stronger the greater the degree to which fundamentals are hard-to-estimate.

[7] It can be shown that Condition (7) is guaranteed to hold eventually almost surely because $\rho\sigma_X\sigma_\delta + \frac{\sigma_X^2}{\lambda}$ is greater than the steady-state level to which the conditional variance, $V^i(t)$, monotonically converges. In addition, note that even though $V^i(0)$ can be large for assets with highly uncertain payoffs, the right hand side of the inequality can also be large for these assets. So Condition (7) can be satisfied by even an asset that has a highly uncertain values before reaching the steady state. With $g^i(u,t) > 0$,

$$\int_0^t g^i(u,t)\log B_u du / \int_0^t g^i(u,t)du,$$

is a weighted moving average of log Bitcoin prices.

momentum (e.g., Moskowitz et al., 2012, and Huang et al., 2019), and the related cross-sectional (CS) momentum phenomenon of Jegadeesh and Titman (1993). One explanation proposed by this literature is sustained over-reaction, which can be caused, for example, by positive feedback trading (DeLong et al., 1990); Hong and Stein (1999), over-confidence and self-attribution confirmation biases (Daniel, Hirshleifer, and Subrahmanyam, 1998), herding (Bikhchandani et al., 1992), or general sentiment (Baker and Wurgler, 2006, 2007). A second explanation is under-reaction, which can be caused, for example, by conservatism bias (Barberis et al., 1998), trend following (Hong and Stein, 1999), and gradual diffusion of information (Hong and Stein, 1999; Hong et al., 2000). In our model, the reaction of investors to signals resembles "under-reaction", but not because of behavioral biases. With hard-to-value fundamentals, investors rationally weight priors and incoming signals that feature uncertainty, which leads to price drift, but this price drift does not represent mispricing.

Moskowitz et al. (2012) find that TS momentum can explain CS momentum empirically and argue that many explanations for CS momentum are really explanations of TS momentum. Thus, our model's setting and prediction seem intuitively related to the finding that CS momentum and post-earnings announcement drift are stronger when information uncertainty is higher (e.g, Zhang, 2006). Presumably, information uncertainty is highly correlated with the degree to which fundamentals are hard-to-value, and therefore we would expect to see greater price drift in segments with high information uncertainty. Moreover, foreign investors, who presumably lack the information of domestic investors, rely relatively heavily on CS momentum strategies (e.g., Choe et al., 1999).

The second implication of Proposition 1 is that the optimal trading strategy is a function of the MAs and trading is driven by a difference in investor beliefs. Thus, our model justifies one of the most widely used class of technical analysis strategies, those based on MAs of prices. No prior rational equilibrium endogenously generates such a practice. For example, early theoretical study of Zhu and Zhou (2009) takes the MA rule as a given strategy for rational investors. Recently, Han et al. (2016) propose a model with technical traders, but the traders use a price-to-moving average ratio rule exogenously. In contrast, our paper seems the first that endogenizes MA trading in the model.

2.3| Cyptocurrency Literature

Our paper contributes to the growing literature on the economics of cryptocurrencies and the associated blockchain technology. Relatively few papers in this vein study asset-pricing properties. Among them, Liu and Tsyvinski (2018) document a TS momentum effect in cryptocurrency returns, but, unlike our paper, do not provide a theory to rationalize this phenomenon. Using the Cagan model of hyperinflation, Jermann (2018) empirically examines the relative contribution of shocks to volume and velocity on variation in Bitcoin's price. Jermann finds that most of the variation in Bitcoin's price is attributable to volume shocks, consistent with stochastic adoption dominating technology innovations. Dwyer (2015) explains how cryptocurrencies can have positive value given limited supply. Athey et al. (2016), Bolt and van Oordt (2016), and Pagnotta and Buraschi (2018) all provide models in which the value of cryptocurrencies depends on some combination of (i) usage and the degree of adoption, (ii) the scarcity of Bitcoin, and (iii) the value of anonymity.

Our model differs from those used by prior studies in at least two important respects. First, our model does not require Bitcoin to be interpreted as a currency per se. We do not directly specify currency-related determinants of its value (e.g. (i)–(iii) above). Rather, we model the flow of utility-providing benefits as a random state variable, which we call a "convenience yield", but admits a more general interpretation. This generality is important because some market participants argue that Bitcoin is better thought of as a speculative asset than a currency (e.g., Yermack, 2013). For example, Bitcoin's high volatility eliminates its use a store of value, a defining feature of money. Second, the papers cited above all assume full information, however, our model features learning. This feature is critical given the lack of agreement on what determines the value of Bitcoin.[8] The learning aspect of our model also helps us to answer novel questions relative to the prior studies such as: What predicts Bitcoin returns?

Relative to asset pricing inquiries like ours, most of the literature on the economics of Bitcoin seeks to identify problems, implementation issues, and uses of cryptocurrencies. Böhme et al. (2015) discuss the virtual currency's potential to disrupt existing payment systems and perhaps even monetary systems. Harvey (2017) describes immense possibilities for the future for Bitcoin and its underlying blockchain technology. Balvers and McDonald (2018) describe conditions and practical steps necessary for using blockchain

[8] For instance, in a Bloomberg interview on December 4, 2013, Alan Greenspan stated: "You have to really stretch your imagination to infer what the intrinsic value of Bitcoin is. I haven't been able to do it. Maybe somebody else can."

technology as a global currency. Easley et al. (2019) provide a model of Bitcoin trading fees. Yermack (2017) discusses use of Blockchain for trading equities and the corresponding governance implications. Gandal et al. (2018) and Griffin and Shams (2018) document Bitcoin price manipulation. Biais et al. (2018) model the reliability of the blockchain mechanism. Catalini and Gans (2017) discuss how blockchain technology will shape the rate and direction of innovation. Chiu and Koeppl (2017) study the optimal design of cryptocurrencies and assess quantitatively how well such currencies can support bilateral trade. Cong and He (2018) model the impact of blockchain technology on information environments. Fernández-Villaverde and Sanches (2017) model competition among privately issued currencies. Foley et al. (2018) document that a large portion of Bitcoin transactions represents illegal activity. Huberman et al. (2017) model fees and self-propagation mechanism of the Bitcoin payment system. Malinova and Park (2017) model the use of blockchain in trading financial assets. Saleh (2017) examines economic viability of blockchain price-formation mechanism. Prat and Walter (2016) show theoretically and empirically that Bitcoin prices forecast Bitcoin production. Krueckeberg and Scholz (2018) argue that Bitcoin may qualify as a new asset class.

3| DATA

Bitcoin trades continuously on multiple exchanges around the world. We obtain daily Bitcoin prices from the news and research site Coindesk.com, which is now standard in academic and professional publications such as the *Wall Street Journal*, over the sample period July 18, 2010 (first day available) through June 30, 2018. Starting July 1, 2013, Coindesk reports a Bitcoin price equal to the average of those listed on large high-volume high-liquidity exchanges. Prior to July 2013, Coindesk reported the price from Mt. Gox, an exchange that handled most of the trading volume in Bitcoin at the time.[9] We also obtain data on two other cryptocurrencies, Ripple (XRP) and Ethereum (ETH), from coinmarketcap.com. These two currencies are the largest competitors to Bitcoin by market cap, but are only available over shorter samples (August 4, 2013–June 30, 2018 for XRP, and August 8, 2015–June 30, 2018 for ETH).

We obtain daily risk-free rate, market excess return (MKT), and returns on three Fama and French (1993) size portfolios from the website of Kenneth French. The three size portfolios, “small", “medium",

[9] For details on the history of the Bitcoin market, see Eha (2017).

and “big", are based on the NYSE 30%- and 70%-iles of market capitalization at the end of each month. To measure the risk-free rate on weekends, we use the most recently available one-day risk-free rate. The average risk free rate over this time (see below) is multiple orders of magnitude smaller than the average Bitcoin return over this time so our risk-free rate assumptions can not have an economically meaningful impact on our results. We obtain individual stock data from CRSP and analyst coverage from IBES. We obtain daily prices and total returns on the NASDAQ total return index from Bloomberg. We obtain daily levels of the CBOE implied volatility index (VIX), 3-month and 10-year Treasury yields ($BILL$ and LTY, respectively), and Moody’s BAA- and AAA-bond index yields (BAA and AAA, respectively) from the St. Louis Federal Reserve Bank website over the sample period July 18, 2010–June 30, 2018. We define TERM = LTY −BILL and DEF = BAA−AAA. VIX, BILL, TERM, and DEF are commonly used returns predictors and among the few available at the daily frequency (e.g., Ang and Bekaert, 2007; Goyal and Welch, 2008; Brogaard and Detzel, 2015).

Table 1 presents summary statistics for select variables used in our predictability tests. Panel A shows that Bitcoin earns an annualized daily excess return of 193.2% and a Sharpe ratio of 1.8 with an annualized volatility of 106.2%. Moreover, Bitcoin has a modest positive autocorrelation. In contrast, MKT has a modest negative autocorrelation and much lower average return and volatility over the period of 13.7% and 14.8%, respectively. Although far less than the Sharpe ratio of Bitcoin, the resulting MKT Sharpe ratio of 0.92 is relatively high by historical standards. Panel B presents summary statistics for several benchmark return predictor variables used in the next section. All four are highly persistent, with an autoregressive coefficient of 0.95–1.0. Moreover, Augmented Dickey-Fuller tests fail to reject the null that any of the return predictors except VIX contain a unit root.

4| EMPIRICAL RESULTS

In this section, we test the predictions from our model in Section 2 that (i) short-horizon returns on Bitcoin and other assets with hard-to-value fundamentals exhibit price drift and are predictable by moving averages of price, and (ii) moving averages can explain Bitcoin trading volume. We begin by examining the predictability of returns on Bitcoin, whose fundamentals are arguably “hardest" to value among the assets we consider.

4.1 In-sample predictability of Bitcoin returns

Motivated by Eqs. (3) and (6), we test the predictability of one-day returns using the difference between the log price of Bitcoin averages (instead of the g^i-based weights).[10] and the moving average of these log prices. For empirical work, we make two simplifications to the moving averages in Eq. (6). First, due to the difficulties of estimating the exact functionals, we assume equal weighting in the moving Second, following Brock et al. (1992), Lo et al. (2000), Han et al. (2013), Neely et al. (2014), and Han et al. (2016), we specify fixed time horizons of L =1, 2, 4, 10, and 20 weeks for the moving averages even though these horizons are endogenous in our model.

Specifically, letting B_t denote the price of Bitcoin on day t, we define:

$$b_t = log(B_t), \tag{9}$$

and the moving averages by:

$$ma_t(L) = \left(\frac{1}{n \cdot L}\right) \sum_{l=0}^{n \cdot L-1} b_{t-l}, \tag{10}$$

where n denotes the number of days per week in L weeks. Bitcoin trades 7 days per week, however stock returns and the macro predictors are only available on the 5 business days per week. Hence, for tests using stock returns and the latter predictors, we use $n = 5$. For tests using only Bitcoin returns and moving averages, we use 7-day-per-week observations ($n = 7$).[11] The log price-to-moving average ratios, denoted $pma_t(L)$, serve as our central predictor of interest in empirical tests and are defined as:

$$pma_t(L) = b_t - ma_t(L). \tag{11}$$

Under condition (7), the $pma_t(L)$ should positively predict Bitcoin returns over short time horizons. Table 2 evaluates in-sample predictive regressions of the form:

[10] The exact functional form is determined by our exact setting, e.g., log utility. The functional form we use in empirical tests still captures the essence of our model, which is robust to more general settings, that price drift exists by functions of past prices, and these functions at least approximate price-to-moving average ratios.

[11] To be clear, using 5-day (7-day) per week observations, the moving average horizons for L =1, 2, 4, 10, and 20 weeks are, respectively, 5, 10, 20, 50, and 100 (7, 14, 28, 70, and 140) days.

$$r_{t+1} = a + b'X_t + \varepsilon_{t+1}, \tag{12}$$

where r_{t+1} denotes the return on Bitcoin on day $t+1$. To facilitate comparison with predictability by $BILL$, $TERM$, DEF, and VIX, we use 5-day "business" weeks throughout the table. Columns (1)–(5) of Panel A present results with $X_t = pma_t(L)$ for each L. The $pma_t(L)$ significantly predict r_{t+1} for all L with the positive sign predicted by our model. The moving averages of different horizons will mechanically be highly correlated with each other. Hence, to test whether different horizons' $pma_t(L)$ contain non-redundant predictive information, column (6) presents results in which the predictors are the first three principal components of the $pma_t(L)$, denoted $X_t = (PC1_t,\ PC2_t,\ PC3_t)'$. The second and third principal components each load with at least marginal significance and the adjusted R^2 is roughly three to four times as high as the specifications in columns (1)–(5). Hence, it appears the set of all $pma_t(L)$ contain at least two distinct predictive signals, consistent with our model in which different traders use different MA horizons.

Panel B presents predictive regressions of the form Eq. (12) using the common "macro" return predictors $X_t = VIX_t$, $BILL_t$, $TERM_t$, or DEF_t. Columns (1)–(5) show that none of these variables significantly predict Bitcoin returns in Eq. (12) either individually or jointly. Moreover, column (6), which uses predictors $X_t = (VIX_t,\ BILL_t,\ TERM_t,\ DEF_t,\ PC1_t,\ PC2_t,\ PC3_t)'$, shows that the macro return predictors do not subsume the predictive power of the $pma_t(L)$.

Condition (7) will hold after enough time elapses with probability one as agents learn and posterior variance decreases. However, at times when the variance of the conditional expectation is relatively high, the predictive coefficient (analogous to the $f^i(t,t) - f^i(0,t)$ in Eq. (6)) on the $pma_t(L)$ should be relatively low. When this variance is high enough to violate condition (7), which is most likely to happen at the beginning of the sample, the predictive coefficient will even become negative. To test these patterns, we proxy for variance of the state variable using a measure of the conditional variance of the Bitcoin return. Specifically, we use the exponentially weighted moving average variance of Bitcoin returns, denoted σ_t^2.[12]

[12] We use the smoothing parameter of 0.94 which is the default from RiskMetrics for computing conditional variances of daily returns. σ_t^2 is defined recursively as $\sigma_t^2 = (0.94) * \sigma_{t-1}^2 + (0.06) * r_t^2$, where r_t is the day-t return on Bitcoin. σ_0^2 is defined to be the sample variance over the first 140 days of our sample that are not used in our return-

Table 3 presents predictive regressions of the form:

$$r_{t+1} = a + b \cdot pma_t(L) + c \cdot \sigma_t^2 + d \cdot pma_t(L) \cdot \sigma_t^2 + \varepsilon_{t+1}. \tag{13}$$

For these regressions, we use the whole sample period 12/06/2010–06/30/2018 and 7-day-per-week observations. The $pma_t(L)$ load significantly for all moving average horizons. Consistent with our model, the interaction terms between $pma_t(L)$ and σ_t^2 are all negative, so high variance attenuates the predictive coefficients on the $pma_t(L)$. Moreover, the interaction terms are significant for three of the five moving average horizons.

The top graph in Figure 1 plots the conditional variance of the Bitcoin returns over time. Consistent with the role of learning in our model, the variability of the conditional variance decreases over time. The bottom graph in Figure 1 plots the coefficient on the $pma_t(4)$ conditional on variance $(b + d \cdot \sigma_t^2)$. Consistent with our model, this coefficient is positive most of the time, especially later in the sample, however it is negative when conditional variance is high enough, early in the sample.

Overall, the in-sample predictability evidence in Tables 2 and 3 is consistent with our model. The $pma_t(L)$ positively predict Bitcoin returns on average. However, high conditional variance that exists before agents have a chance to "learn it away" can reverse this predictive relationship.

4.2| Out-of-sample predictability of Bitcoin returns

It is well established that highly persistent regressors such as VIX, $BILL$, $TERM$, and DEF can generate spuriously high in-sample return predictability (e.g., Stambaugh, 1999; Ferson et al., 2003; Campbell and Yogo, 2006). These biases, parameter instability, and look-ahead biases imply that in-sample estimates can overstate true real-time predictability, which directly impacts investors (e.g., Goyal and Welch, 2008). Hence, we next assess the out-of-sample predictability of Bitcoin returns.

Table 4 presents out-of-sample R^2 (R^2_{OS}) of forecasts from recursively estimated regressions similar to those estimated in-sample in Table 2.[13] The first five columns of Panel A report R^2_{OS} based on

prediction tests because they are required to compute the initial 140-day moving average. In particular, the σ_t^2 is not based on any "in-sample" data used in the predictive regressions. In our model, it can be shown that the conditional variance of *discretized* returns is tightly linked to the posterior variance.

[13] All out-of-sample regressions in this Table use expanding (not rolling) windows using all data available through t to make the forecast for day $t+1$.

regressions of the form Eq. (12). For robustness, we report R_{OS}^2 using several split dates between the in-sample and out-of-sample periods that include both relatively large in- and out-of-sample periods (e.g., Kelly and Pruitt, 2013). The last column (denoted MEAN), follows Rapach et al. (2010) and presents R_{OS}^2 for the MEAN combination forecast, which is the simple average of the forecasts from the first five columns. Prior studies find that the MEAN combination forecasts are robust, frequently outperforming more sophisticated combination methods (that have more estimation error) in forecasting returns and other macroeconomic time-series out-of-sample (e.g., Timmermann, 2006; Rapach et al., 2010; Detzel and Strauss, 2018). Moreover, with diffuse priors about which MA horizon is optimal, technical traders would presumably give equal-weight to the different forecasts.

Panel A shows that several of the $pma_t(L)$ individually predict returns out-of-sample with $R_{OS}^2 > 0$. Moreover, for each split date, the MEAN forecasts forecasts predict returns with at least marginal significance and R_{OS}^2 of 0.83%–1.42%, which are high for the daily horizon. For comparison, Pettenuzzo et al. (2014) find out-of-sample R^2 ranging from -0.08% to 0.55% for monthly stock returns. Panel B presents results from similar tests as Panel A, but using VIX, $BILL$, $TERM$, and DEF as predictors. Unlike the forecasts based on the $pma_t(L)$, those based on the macro predictors generally have negative R_{OS}^2. Prior evidence show that predicting returns out-of-sample is challenging, especially at short horizons. Hence, it is already remarkable that we observe one-day out-of-sample predictability of Bitcoin returns by the $pma_t(L)$. It should also be the case that this predictability increases with horizon. Thus, in Panel C, we present R_{OS}^2 based on recursively estimated regressions of one-week (7-day) Bitcoin returns on the $pma_t(L)$:

$$r_{t+1,t+7} = a + b \cdot pma_t(L) + \varepsilon_{t+1,t+7}. \tag{14}$$

Consistent with prior evidence on stock and bond return predictability, Panel C shows that for each out-of-sample window and each L, the R_{OS}^2 generally increase in both magnitude and significance relative to the analogous one-day-return R_{OS}^2 in Panel A. The MEAN forecast, for example, has R_{OS}^2 that are statistically significance and large for weekly returns. For comparison, Rapach et al. (2010) find R_{OS}^2 of 1%–3.5% for quarterly stock returns. It is also worth noting that the predictability is not confined to the early part of the sample, the most recent 10% of the sample still has large and statistically significant R_{OS}^2. Overall, the out-of-sample evidence shows that the in-sample predictability of Bitcoin returns does not represent small-sample biases and evinces that investors can take advantage of Bitcoin predictability by moving averages of log prices.

4.3| Performance of Bitcoin technical analysis strategies

The results above show that the $pma_t(L)$ predict Bitcoin returns with statistical significance. Next, we evaluate the associated economic significance by assessing the performance of trading strategies based on this predictability predictability (e.g., Pesaran and Timmermann, 1995; Cochrane, 2008; Rapach et al., 2010). We define the buy indicator (buy=1) associated with each MA strategy, MA(L), as:

$$S_{L,t} = \begin{cases} 1, \text{if} \quad pma_t(L) > 0 \\ 0, \ \text{otherwise} \end{cases} \tag{15}$$

The return on the Bitcoin MA(L) strategy on day t is given by:

$$r_t^{MA(L)} = S_{L,t} \cdot r_t + (1 - S_{L,t}) \cdot r_{ft}, \tag{16}$$

where r_t and r_{ft} denote, respectively, the return on Bitcoin and the risk-free rate on day t. Intuitively, the trading strategy defined by Eq. (16) captures the short-term trends predicted by our model by going long Bitcoin when its price is expected to trend upward, and vice versa. We denote the excess return of the buy-and-hold position in Bitcoin as rx_t and the excess return on the MA(L) strategies by $rx_t^{MA(L)}$.

Table 5 presents summary statistics for the buy-and-hold and MA(L) strategies. Panel A, which uses the full sample (12/06/2010–6/30/2018), shows that all strategies are right-skewed and have fat tails. The Sharpe ratio of Bitcoin is 1.8, which is about four times the historical Sharpe ratio of the stock market (e.g., Cochraine, 2005). All of the MA(L) strategies further increase this ratio to 2.0 to 2.5. Moreover, all but one of these Sharpe ratio gains are at least marginally significant using the heteroskedasticity and autocorrelation (HAC) robust test for equality of Sharpe ratios of Ledoit and Wolf (2008). The maximum drawdown of Bitcoin is 89.5%, while those of the MA(L) strategies are all lower, ranging from 64.4% to 77.9%.

Comparing Panels B and C indicates that the performance of Bitcoin was higher during the first half of the sample, although the Sharpe ratio gains of the MA(L) strategies relative to the buy-and-hold position are similar in both subsamples.

Panel A of Figure 0 plots the cumulative value of $1 invested in Bitcoin and the MA(2) (two-week) strategy at the beginning of the sample. At the end of our sample, the $1 in Bitcoin grew to $33,617 while the $1 in the MA$(2)$ strategy grew to approximately $148,549, a difference of about $114,932 over 7.5 years! Panel B plots the drawdowns of Bitcoin and the MA(2) strategy. As Panel B shows, the out-performance of the MA strategies relative to the buy and hold largely stems from the MA strategy having both shorter and less severe drawdowns than the buy-and-hold. For example, Bitcoin prices hit an all-time high in December 2017 at $19,343 and subsequently fell to $6,343 by the end of our sample. Panel B shows investors using the MA(2) strategy would have been spared most of the losses from this price decline.

Table 6 further tests the performance of MA strategies relative to the buy-and-hold. Specifically, we regress the excess returns of the MA strategies on the buy-and-hold benchmark:

$$rx_t^{MA(L)} = \alpha + \beta \cdot rx_t + \varepsilon_t. \tag{17}$$

The MA(L) strategies are long the risk-free rate up to about 40% of days, so $\beta < 1$ and $\beta \cdot rx_t$ is the natural benchmark return for evaluating the average returns of $rx^{MA(L)}$. Moreover, the arguments in Lewellen and Nagel (2006) show that α will increase with the quantity $\mathrm{cov}(S_{L,t}, E_t(rx_t))$, which measures the degree to which the $pma_t(L)$ positively predicts Bitcoin returns.

A positive alpha also indicates that access to $rx_t^{MA(L)}$ increases the maximum possible Sharpe ratio relative to that of a buy-and-hold Bitcoin position (rx_t). Thus, a measure of the economic size of alpha is the degree to which it expands the mean-variance frontier. Intuitively, this expansion depends on the alpha relative to the residual risk investors must bear to capture it. The maximum Sharpe ratio (SR_{New}) attainable from access to rx_t and $rx_t^{MA(L)}$ is given by:

$$SR_{New} = \sqrt{\left(\frac{\alpha}{\sigma(\varepsilon_t)}\right)^2 + SR_{Old}^2}, \tag{18}$$

where SR_{Old} is the Sharpe ratio of rx_t (e.g., Bodie et al., 2014). The percentage increase in mean-variance utility, which, for any level of risk aversion, is equal to:

$$\text{Utility} \quad \text{gain} = \frac{SR_{New}^2 - SR_{Old}^2}{SR_{Old}^2}, \tag{19}$$

measures the economic significance of the frontier expansion achieved by α. In Table 6, we report both the appraisal ratio $\left(\frac{\alpha}{\sigma(\varepsilon_t)}\right)$ and mean-variance utility gains along with the α. For comparison, Campbell and Thompson (2008) find that timing expected returns on the stock market increases mean-variance utility by approximately 35%, providing a useful benchmark utility gain.

Panel A shows that over the entire sample period, the MA(L) strategies earn significant α with respect to rx_t of 0.09% to 0.24% per day. These alphas lead to economically large utility gains of 19.7% to 85.5%. Panel B shows these results remain strong in the second half of the sample. With the turnovers in the Table, it would take large transaction costs of (1.38%–6.85% one-way) to eliminate the alphas of the MA(L) strategies. These figures are large relative to actual one-way transaction costs in Bitcoin. For example, market orders on the Bitcoin exchange GDAX have fees of 0.10%–0.30% for market orders and 0% for limit orders. Even the most expensive market order fees are an order of magnitude too small to meaningfully impact the αs of the MA(L) strategies.

A naive alternative to our discrete buy-or-sell strategies defined by Eq. (16) would be estimating mean-variance weights using our Bitcoin-return forecasts, and then testing whether the resulting strategy out-performs the buy-and-hold benchmark (e.g., Marquering and Verbeek, 2004; Campbell and Thompson, 2008; Huang et al., 2015). However, this approach has several theoretical and empirical shortcomings relative to our simple MA(L) strategies. First, the mean-variance weights assume the investor is choosing between the market return and the risk-free asset. However, Bitcoin is a poor theoretical proxy to the market portfolio of risky assets. Second, prior to 2017, investors could not short-sell Bitcoin or buy it on margin or via futures contracts. Hence, the weights on Bitcoin should be constrained between zero and one. Thus, the mean-variance-weights approach could only outperform the MA strategies by choosing optimal variation between zero and one. This in turn exacerbates the following two problems: (i) that the mean-variance weights require at least two estimated forecasts, and therefore come with substantial estimation error, and

(ii) the mean-variance weights assume for tractability the mean-variance functional form of investor utility. While a common assumption, mean-variance utility is unlikely to precisely capture the behavior of a representative investor. In contrast, our discrete $\mathrm{MA}(L)$ strategies are based on a directly observable out-of-sample signals and require no estimation error. They also make no assumption about the utility of underlying investors. Overall, the strong performance of our $\mathrm{MA}(L)$ strategies relative to the buy-and-hold precludes the need for more sophisticated methods to demonstrate the economic significance of out-of-sample predictability by MAs.

4.4| Performance of trading strategies applied to other cryptocurrencies

To examine the robustness of our trading strategy performance, Table 7 presents performance results similar to those above for Ripple (XRP, Panels A and B) and Etheruem (ETH, Panels C and D), which are the two largest digital currencies by market capitalization beside Bitcoin. Panel A shows that all the MA strategies except $\mathrm{MA}(4)$ increase Sharpe ratios relative to the buy-and-hold strategy by up to 0.54 (from 1.05). This difference is significant for the $\mathrm{MA}(1)$ and $\mathrm{MA}(2)$ strategies and marginally significant for the equal-weighted portfolio of MA strategies (EW). Each strategy reduces the maximum drawdown of the buy-and-hold Ripple strategy by about 4.7%-32.3%. Panel B shows that the $\mathrm{MA}(1)$, $\mathrm{MA}(2)$, $\mathrm{MA}(4)$, and EW strategies also earn significant alphas with respect to the buy-and-hold XRP strategy, generating large utility gains (92.9%–180.1%) in the process.

Panels C and D present similar results as Panels A and D, respectively, but for strategies based on ETH instead of XRP. The ETH sample is only two and a half years long, leading to relatively low statistical power, but qualitatively similar inferences as for the Bitcoin and Ethereurm MA strategies. The MA strategies earn higher Sharpe ratios than the buy-and-hold ETH strategy. Panel C shows the ETH MA-strategy alphas are significant for three horizons (1, 2, and 4 weeks) as well as the EW strategy and the associated utility gains are economically large.

4.5| Performance of strategies applied to dotcom-era NASDAQ portfolio

Next, we apply each of the MA strategies defined by Eq. (16) to the NASDAQ total return index using daily data over the sample 1996–2005, a ten year window approximately centered around the peak of the NASDAQ “bubble" in March 2000. During this time, fundamentals of tech stocks were difficult to interpret, widely disagreed upon, and had great forecast uncertainty. For example, Ofek and Richardson (2003) document that during this period, aggregate earnings of internet stocks were negative and price-earnings ratios frequently exceeded 1000. In particular, NASDAQ fundamentals plausibly qualify as hard-to-interpret in the 1996–2005 sample.

Table 8 documents the performance of the MA strategies applied to the NASDAQ. Results in Panel A show that over 1996–2005, the $MA(2)$, $MA(4)$ and $MA(10)$ methods possess mean returns more than four percent greater than the 7.3% of the buy-and-hold NASDAQ strategy. Further, all five methods substantially boost the NASDAQ Sharpe ratio of 0.29. For example, $MA(2)$, $MA(4)$ and $MA(10)$ possess Sharpe ratios of 0.73 to 0.79. The last column documents that the MA strategies also greatly reduce the maximum drawdown of NASDAQ (77.9%) to 25.7%–45.6%.[14] Panel B of Table 8 presents the alphas, appraisal and utility gain for the NASDAQ. Results document significant alpha for $MA(2)$ to $MA(10)$ strategies. It also reveals high utility gains for all five strategies, ranging from 137%–688%.

Panel C of Table 8 presents results over a tighter window around the NASDAQ peak. A number of new internet companies entered the NASDAQ around this period, and the fundamentals of many other firms were questioned after several large earnings misstatements. The NASDAQ peaked in March 2000, and by the end of 2002 had lost 78% of its value. In contrast, the MA strategies have maximum drawdowns from 34%-43% and the equal-weighted MA strategy lost only 34% of its value—less than half the buy-and-hold position. Further, the Sharpe ratios for the MA strategies during this five year window ranged from 0.36-0.60, compared to zero for the buy-and-hold. The equal-weighted strategy generates a Sharpe ratio of 0.57 and is significantly greater than the buy-and-hold Sharpe ratio. Overall, the performance of the trading strategies documented in Table 8 is consistent with the predictions of our model.

Figure 3 depicts the performance of the buy-and-hold position in NASDAQ relative to the $MA(4)$ strategy. Panel A shows the $MA(4)$ increases more steadily than NASDAQ. The $MA(4)$ returns \$3.66 at

[14] We also applied our strategies to the NASDAQ over the past ten years, which follows the maturation of internet-based technologies and an increased understanding of fundamentals. These untabulated results show that the MA strategies no longer earn significant alpha or produce Sharpe ratio gains.

the end of 2005 to an investor with a $1 investment at the beginning of 1996. Conversely, a buy-and-hold investor in NASDAQ would have about half ($1.85) of the accumulated value. Panel A shows that much of the performance gains from the MA strategy come from avoiding most of NASDAQ's large crash in the early 2000's. Panel B further shows that the MA(4) strategy, similar to when applied to Bitcoin, derives much of its performance from avoiding the major NASDAQ drawdowns during this time period. Following the dotcom era, tech companies become more established and the availability of value-relevant information presumably increases. Hence, according to our model, the MA strategies performance should decline after this time period. Panel C confirms that, indeed, the Sharpe ratio improvements of the NASDAQ MA(4) strategy steadily decline post-2001.

4.6| Performance of strategies applied to small-cap stocks, young stocks, and stocks with low analyst coverage

The NASDAQ results exploit time-series variation in the degree to which fundamentals are hard to forecast. Next we exploit cross-sectional variation. Specifically, we examine the predictability of returns by price-to-MA ratios across portfolios sorted on size, age, and analyst coverage, which are three common proxies for the availability of value relevant information (e.g., Bhushan, 1989; Hong et al., 2000; Zhang, 2006).

In Panels A and B of Table 9, we apply our MA strategies to each of the three value-weighted Fama and French (1993) size portfolios, "Small", "Medium" and, "Big". The sample period is July 1, 1963 through June 30, 2018.[15] Panel A presents heteroskedasticity-robust t-statistics from regressions of daily excess portfolio returns on the price-to-moving average ratios:

$$rx_{t+1} = a + b \cdot pma_t(L) + \varepsilon_{t+1}. \quad (20)$$

For each MA, these t-statistics are positive and highly significant (3.96–6.03) for the portfolio of small-cap stocks. The t-statistics fall for mid-cap stocks (1.67–4.43) and become negative and insignificant for large-cap stocks ((-0.96)–(-0.10)). Hence, consistent with our model, the predictability of returns by the $pma_t(L)$ is greater for small-cap stocks than large-caps.

[15] Untabulated results show that inferences are robust across each half of this sample as well.

Panel B presents Sharpe ratios for the buy-and-hold (BH) return on each portfolio as well as each of the MA strategies and the equal-weighted portfolio (EW) of the MA strategies. The figures parallel those in Panel A: Sharpe ratio gains of the MA strategies are highest for small-cap stocks, followed by mid-caps, and then large-caps. For example, in small-caps, the Sharpe ratio gain of the EW strategy are 1.91 from 0.5, relative to 0.65 from 0.39 in large caps. While the performance of the MA strategies is higher for small-caps than large caps, the gains are still nontrivial for large-cap stocks. Coupled with the fact that large-cap stocks returns are unpredictable by the $pma_t(L)$, this finding indicates that the $pma_t(L)$ are useful volatility-timing signals for large-caps.[16] Panels C and D repeat the analysis of Panels A and B, respectively, for value-weighted tercile portfolios formed at the end of each June based on firm age, which, following Zhang (2006), is defined to be the number of years a firm is in the CRSP database. Consistent with our theory and the results in Panels A and B, Panels C and D show that younger firms, which are presumably harder to value, have relatively high predictive coefficients in Eq. (20) and relatively high Sharpe ratios from the $pma_t(L)$ strategies. For example, the predictive coefficients for young firms in Panel C are significant for all $pma_t(L)$ with t-statistics of 2.89 or higher, while they are never significant, and often negative, for old firms. Moreover, while the BH Sharpe ratios are the same for old and young firms in Panel D, they MA Sharpe ratios of young firm are about twice or more than those of the old firms.

Table 10 presents t-statistics for predictive coefficients from regressions of the form Eq. (20) for portfolios formed by independent sorts on size and analyst coverage. Due to IBES data availability, the sample period for these tests is January 1985 through June 2018. Consistent with our model, the t statistics decrease with both size and analyst coverage. The t-statistics are large and positive in small caps, and within small caps, predictive coefficients are significantly higher for stocks with the lowest analyst coverage. Conversely, the predictive coefficients are all negative for large caps, and significantly moreso for the large-cap portfolios with high analyst coverage.

Overall, the results from Tables 9 and 10 show that, consistent with our model, the price-to-MA

[16] Sharpe ratio gains and alphas increase with (positive) return predictability of the timing signal (which, in our case, are the $pma_t(L)$) and decrease with (positive) predictability of volatility (e.g., Lewellen and Nagel, 2006). That is, these performance metrics measure the combined effect of “market timing” and “volatility timing” benefits of a given strategy.

ratios predict returns of small-cap, young-firm, and low-analyst-coverage stocks, all of which have a relative lack of value-relevant information, and fail to predict returns of old-firm and large-cap stocks.

4.7| Volume implications of our model

In our model, trading results from differences in the moving averages used by different traders. Testing this refutable implication provides an opportunity to validate our model's mechanism in explaining the predictability of Bitcoin by moving averages of multiple horizons.

We test for volume generated by technical trading in two ways. First, we evaluate whether increases in total turnover implied by different MA signals also leads to higher Bitcoin volume. We measure this total turnover by the sum of the turnover generated by each moving-average buy-sell indicator $S_{L,t}$: $\sum_L |\Delta S_{L,t}|$. Second, we evaluate whether disagreement among MA buy-sell indicators ($S_{L,t}$) is associated with higher trading volume. Intuitively, if technical traders disagree, they will trade with each other. As a measure of disagreement, we use the cross-sectional standard deviation of the signed turnover implied by each MA strategy, denoted $\sigma_L(\Delta S_{L,T})$. For each measure, Table 11 presents estimations of regressions of the form:

$$\Delta \log(volume_t) = a + b \cdot X_t + c \cdot |r_t| + d \cdot \Delta \log(volume_{t-1}) + \varepsilon_t, \tag{21}$$

where the X_t denotes one or both of our two volume-inducing variables. Because large price shocks are the main empirical determinant of volume and are likely correlated with our price-based indicators, we control for the absolute value of returns (see, e.g., Karpoff, 1987). We use change in log volume as the dependent variable because the level of volume is not stationary. Volume is from coinmarketcap.com, which began reporting on 12/27/2013, so these regressions use the 12/27/2013–6/30/2018 ($n = 1{,}647$). In Panel A, we restrict $d = 0$. However, to avoid any possibility of results being driven by autocorrelation in volume, we do not make this restriction in Panel B.

Results in column (1) of both Panels demonstrate that increases in turnover across MA horizons lead to increases in volume, controlling for price shocks. Similarly, column (2) of each panel shows that increases in disagreement among MA traders also leads to significant increases in volume. Finally, comparing column (3) of each Panel shows that the MA-implied turnover and disagreement jointly and positively correlate with volume, however the statistical inference varies with specification.[17] Overall, the results in Table 11 are consistent with traders using MA strategies significantly impacting trading volume in Bitcoin.

5.| CONCLUSION

In this paper, we theoretically and empirically examine dynamics of the prices of assets with "hard-to-value" fundamentals, such as Bitcoin. We propose a new equilibrium theory that shows that when fundamentals are hard to value, rational learning causes price drift and ratios of prices to their moving averages to forecast returns. This in turn provides a fully rational motivation for common technical analysis strategies that use price-to-moving average ratios, which are typically justified by mispricing-based arguments. Our empirical results strongly confirm the predictions of our model. Bitcoin and stocks with hard-to-value fundamentals are predictable by price-to-moving average ratios and simple real-time strategies based on this predictability significantly outperform the buy-and-hold strategy. Given that the key assumption underlying our model is the difficulty of forecasting fundamentals, a potentially fruitful avenue for future research is extending our results to other assets well-described by this assumption, perhaps new asset classes, and to examine the degree to which price drift is concentrated within these assets.

[17] This finding is likely not due to multicollinearity, the correlation between $\sigma_L(\Delta S_{L,t})$ and $\Sigma_L(\Delta S_{L,t})$ is 0.53.

Appendix: Proof of Proposition 1

In this appendix, we present the proof of Proposition 1.

First we provide the evolution equations for conditional expectation and the conditional variance. Following the standard continuous-time filtering theory, $\forall i = 1, 2$, M_t^i satisfies

$$dM_t^i = \lambda(\bar{X} - M_t^i)dt + \sigma_M^i(t)d\hat{Z}_{1t}^i, \quad M_0^i = M^i(0), \tag{1}$$

where $\hat{Z}_{1t}^i$ is the (observable) innovation processes satisfying

$$\hat{Z}_{1t}^i = \int_0^t \frac{X_s - M_s^i}{\sigma_\delta} ds + Z_{1t},$$

$\sigma_M^i(t) = \frac{V^i(t)}{\sigma_\delta} + \rho\sigma_X$, $V^i(t) \equiv E[(X_t - M_t^i)^2 | F_t^i]$ is the conditional variance of X_t satisfying

$$\frac{dV^i(t)}{dt} = -2\lambda V^i(t) + \sigma_X^2 - \left(\frac{1}{\sigma_\delta} V^i(t) + \rho\sigma_X\right)^2. \tag{2}$$

This implies that

$$\frac{d\delta_t}{\delta_t} = M_t^i dt + \sigma_\delta d\hat{Z}_{1t}^i, \quad i = 1,\ 2. \tag{3}$$

Next we derive the optimal trading strategy and equilibrium Bitcoin price. As in Detemple (1986), Gennotte (1986), and Detemple (1991).[18] In particular, given the initial endowment $\eta_i > 0$ and the prior $(M_i(0^-), V_i(0^-))$, Investor i's portfolio selection problem is equivalent to

$$\max_{\theta^i, C^i} E \int_0^T e^{-\beta t} \log C_t^i dt,$$

subject to

$$dW_t = r_t W_t dt + \theta_t^i(\mu_t^i - r_t)dt + \theta_t^i \sigma_\delta d\hat{Z}_{1t}^i - C_t^i dt. \tag{4}$$

Define π_t^i as the state price density for investor i. Then

$$d\pi_t^i = -r_t \pi_t^i dt - \kappa_t^i \pi_t^i d\hat{Z}_{1t}^i, \tag{5}$$

where κ_t^i is the price of risk perceived by investor i, i.e.,

$$\kappa_t^i = \frac{\mu_t^i - r_t}{\sigma_\delta}. \tag{6}$$

Using the standard dual approach (e.g., Cox and Huang, 1989) to solve Investor i's problem, we have

$$e^{-\beta t}(C_t^i)^{-1} = \xi_i \pi_t^i, \quad i = 1,\ 2, \tag{7}$$

[18] The separation principle applies because the objective function is independent of the unobservable state variable (see, e.g., Fleming and Rishel (1975, Chap. 4, Sec. 11) .

where ξ_i is the corresponding Lagrangian multiplier. Define

$$\alpha_t = \frac{\xi_1 \pi_t^1}{\xi_2 \pi_t^2} \tag{8}$$

to be the ratio of the marginal utilities. Then α_t evolves as

$$d\alpha_t = -\alpha_t \frac{\mu_t^d}{\sigma_\delta} d\hat{Z}_{1t}^1, \quad \mu_t^d = \mu_t^1 - \mu_t^2, \quad \alpha_0 = \frac{\eta_2}{\eta_1}, \tag{9}$$

where the first equality is from Ito's lemma and the consistency condition (i.e., the Bitcoin price is the same across all investors), and the last equality follows from the budget constraints.

By market clearing condition $C_t^1 + C_t^2 = \delta_t$, Equation (7) and Equation (8), we have

$$C_t^1 = \frac{\delta_t}{1+\alpha_t}, \quad C_t^2 = \frac{\alpha_t \delta_t}{1+\alpha_t}. \tag{10}$$

Then applying Ito's lemma to (7) and compare with equation (5), we have

$$\kappa_t^1 = \sigma_\delta + \frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta}, \quad \kappa_t^2 = \sigma_\delta - \frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta},$$

and

$$r_t = \beta + \frac{1}{1+\alpha_t} M_t^1 + \frac{\alpha_t}{1+\alpha_t} M_t^2 - \sigma_\delta^2. \tag{11}$$

Therefore, the fraction of wealth invested in the Bitcoin by Investor 1 is

$$\kappa_t^1 / \sigma_\delta,$$

i.e.,

$$1 + \frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta^2}, \tag{12}$$

and by Investor 2 is

$$1 - \frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta^2}. \tag{13}$$

So if $\mu_t^d > 0$, i.e., Investor 1 is more optimistic than Investor 2, then Investor 1 borrows to buy the Bitcoin, and Investor 2 sells the Bitcoin and lends.

Using the expression for C_t^1 and equation (7), we have the Bitcoin price

$$B_t = E_t^1 \int_t^T \frac{\pi_s^1}{\pi_t^1} \delta_s ds = \frac{1-e^{-\beta(T-t)}}{\beta} \delta_t,$$

which implies that

$$dB_t = ((\beta + M_t^i)B_t - \delta_t)dt + \sigma_\delta B_t d\hat{Z}_{1t}^i,$$

$$\mu_t^i = \beta + M_t^i. \tag{14}$$

This implies that

$$d\hat{Z}_{1t}^{i} = \frac{1}{\sigma_\delta}\left(d\log B_t - \left(M_t^i - \frac{\beta}{1-e^{-\beta(T-t)}} - \frac{1}{2}\sigma_\delta^2\right)dt\right).$$

Investor 1's wealth is

$$W_{1t} = E_t^1 \int_t^T \frac{\pi_s^1}{\pi_t^1} C_s^1 ds = \frac{1-e^{-\beta(T-t)}}{\beta} C_t^1 = \frac{1}{1+\alpha_t} B_t$$

and Investor 2's wealth is

$$W_{2t} = E_t^1 \int_t^T \frac{\pi_s^2}{\pi_t^2} C_s^2 ds = \frac{1-e^{-\beta(T-t)}}{\beta} C_t^2 = \frac{\alpha_t}{1+\alpha_t} B_t.$$

The number of Bitcoin Investor 1 holds is equal to

$$N_{1t} = \frac{(1+\frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta^2})W_{1t}}{B_t} = \frac{1}{1+\alpha_t}(1+\frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta^2}).$$

The number of Bitcoin Investor 2 holds is equal to

$$N_{2t} = \frac{(1-\frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta^2})W_{2t}}{B_t} = \frac{\alpha_t}{1+\alpha_t}(1-\frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma_\delta^2}).$$

We have thus

$$\frac{\partial N_{1t}}{\partial \alpha_t} = \frac{-(1+\alpha_t)+(1-\alpha_t)\mu_t^d/\sigma_\delta^2}{(1+\alpha_t)^3},$$

which is < 0 if and only if

$$\alpha_t > \frac{\mu_t^d/\sigma_\delta^2 - 1}{\mu_t^d/\sigma_\delta^2 + 1}.$$

Next we derive the expression of the conditional expectation M_t^i in the form of moving averages. We have

$$dM_t^i = (a^i(t) - b^i(t)M_t^i)dt + c^i(t)d\log B_t, \tag{15}$$

where

$$a^i(t) = \lambda\bar{X} + \left(\frac{\beta}{1-e^{-\beta(T-t)}} + \frac{1}{2}\sigma_\delta^2\right)c^i(t),$$

$$b^i(t) = \lambda + c^i(t), c^i(t) = \frac{\sigma_M^i(t)}{\sigma_\delta}.$$

Equation (15) implies that

$$M_t^i = h^i(t) + \int_0^t f^i(u,t)d\log B_u,$$

where

$$h^i(t) = e^{-\int_0^t b^i(s)ds}\int_0^t a^i(u)e^{\int_0^u b^i(s)ds}du,.$$

and

$$f^i(u,t) = c^i(u) e^{\int_t^u b^i(s)ds}. \tag{16}$$

Then by integration by parts, we have

$$M_t^i = h^i(t) - f^i(0,t)\log B_0 + c^i(t)\log B_t - \int_0^t \log B_u df^i(u,t)$$

$$= h^i(t) + f^i(0,t)\log\frac{B_t}{B_0} + (f^i(t,t) - f^i(0,t))\left(\log B_t - \frac{\int_0^t g^i(u,t)\log B_u du}{\int_0^t g^i(u,t)du}\right),$$

where

$$g^i(u,t) = \frac{\partial f^i(u,t)}{\partial u}. \tag{17}$$

We show next that if Condition (7) is satisfied, then $g^i(u,t) > 0$. To prove this, we substitute $f^i(u,t)$ in Equation (16) into (17) to obtain

$$g^i(u,t) = \left(\frac{dc^i(u)}{du} + c^i(u)b^i(u)\right) e^{\int_t^u b^i(s)ds}, \tag{18}$$

thus we need to find condition for $\frac{dc^i(u)}{du} + c^i(u)b^i(u) > 0$. Note that

$$\frac{dc^i(t)}{dt} = -2\lambda\left(c^i(t) - \rho\frac{\sigma_X}{\sigma_\delta}\right) + \frac{\sigma_X^2}{\sigma_\delta^2} - (c^i(t))^2, \tag{19}$$

and $b^i(t) = \lambda + c^i(t)$, we need to have the following condition

$$c^i(t) < 2\rho\frac{\sigma_X}{\sigma_\delta} + \frac{\sigma_X^2}{\lambda\sigma_\delta^2}. \tag{20}$$

Due to the dynamics of $c^i(t)$ given in Equation (19), it can be proven that at $c^i(t) = 2\rho\frac{\sigma_X}{\sigma_\delta} + \frac{\sigma_X^2}{\lambda\sigma_\delta^2}$, $\frac{dc^i(t)}{dt} < 0$. This implies that as long as

$$c^i(0) \le 2\rho\frac{\sigma_X}{\sigma_\delta} + \frac{\sigma_X^2}{\lambda\sigma_\delta^2}, \tag{21}$$

or equivalently,

$$V^i(0) \le \rho\sigma_X\sigma_\delta + \frac{\sigma_X^2}{\lambda}, \tag{22}$$

Condition (7) holds.

Under Condition (7), the expression

$$\frac{\int_0^t g^i(u,t)\log B_u du}{\int_0^t g^i(u,t)du} \tag{23}$$

is a weighted average of $\log(B_u)$ over the interval $[0,t]$. In addition, by the definition of $g^i(u,t)$, this implies that

$$f^i(t,t) - f^i(0,t) > 0$$

for any t. This completes the proof of Proposition 1.

Figure 1: Conditional variance of Bitcoin returns

Panel A depicts the condition variance of Bitcoin returns using an exponentially weighted moving average variance (σ^2) of daily Bitcoin returns.

Panel B depicts the predictive coefficient of $pma_t(4)$ conditional on $(b+d\cdot\sigma^2)$ using the following regression estimated in Table 3:

$$\boldsymbol{r_{t+1} = a + b \cdot pma_t(L) + c \cdot \sigma_t^2 + d \cdot pma_t(L) \cdot \sigma_t^2 + \varepsilon_{t+1}}. \quad (1)$$

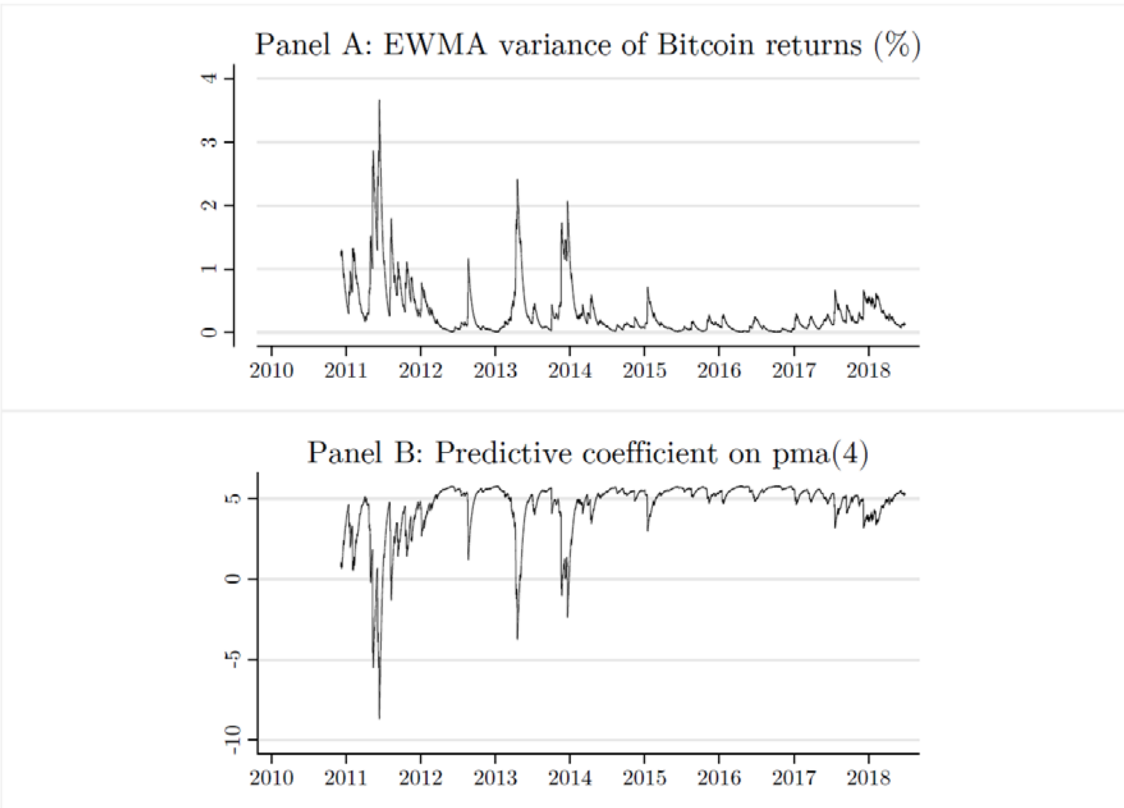
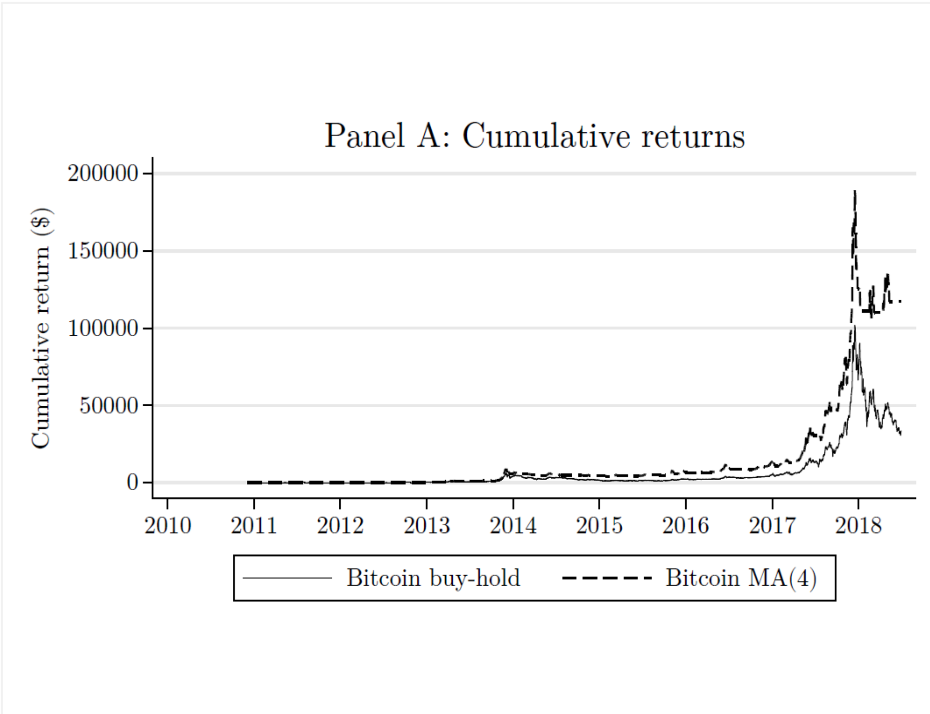


Figure 2: Performance of investment in Bitcoin buy-and-hold and MA strategies

Panel A presents cumulative returns to $1 invested in the buy-and-hold and MA(4) Bitcoin strategies over 7/18/2010–6/30/2018. Panel B presents drawdowns of each strategy in Panel A.



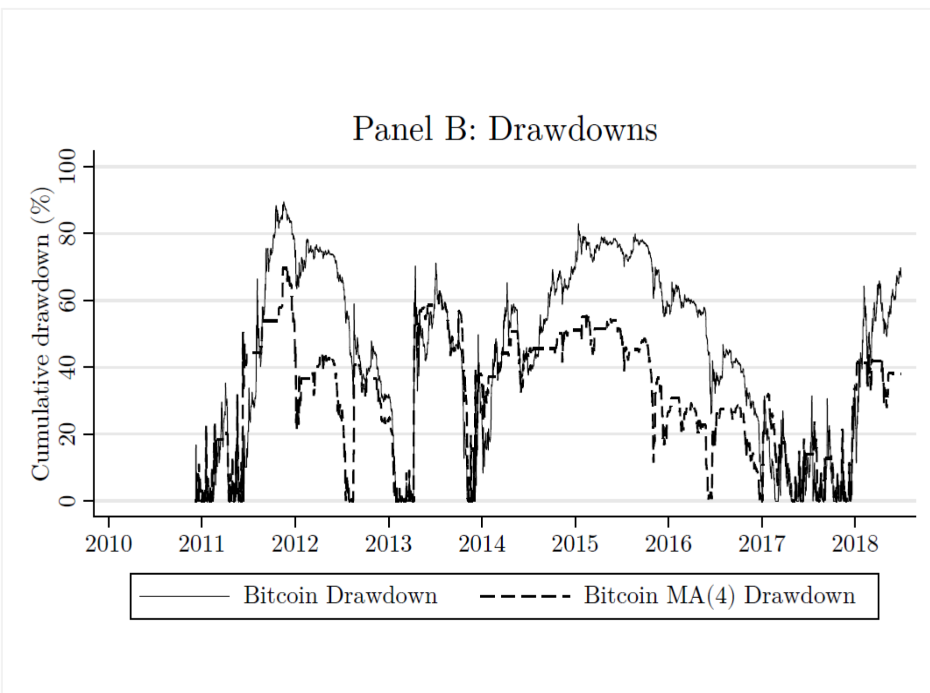


Figure 3: Performance of investment in NASDAQ buy-and-hold and MA(4) strategies

Panel A plots cumulative returns to $1 invested in the buy-and-hold and MA(4) NASDAQ strategies on 1/2/1996 through 12/30/2005. Panel B plots drawdowns of each strategy. Panel C plots Sharpe ratios for each strategy estimated, for each date *t*, using the sample period 1/2/1996 through *t*.

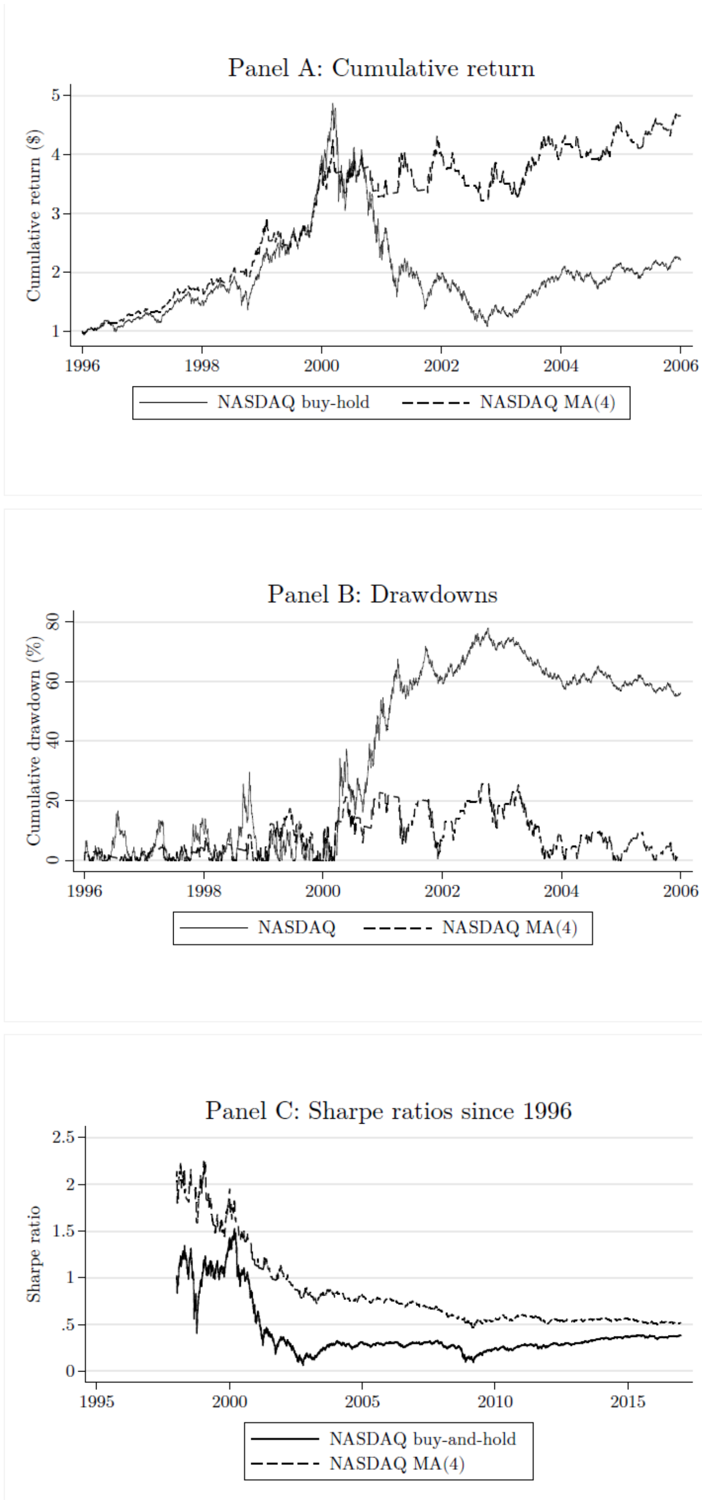


Table 1: Summary statistics

Panel A presents summary statistics of the returns in excess of the 1-day risk-free rate on Bitcoin (BTC) and the CRSP value-weighted index (MKT). Means, standard deviations, and Sharpe ratios are annualized. Panel B presents summary statistics of other relevant variables. AR1 denotes the first-order autoregressive coefficient and p_{df} denotes the p-value from an augmented Dickey-Fuller test for the null of a unit root. The sample period is daily from 12/06/2010−6/30/2018. Bitcoin returns trade 7 days a week and have 2,766 observations during the sample period. Other variables are available 5 days a week and have 1,976 observations during this period.

Panel A: Returns								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	AR1
BT	193.18	106.20	1.82	-38.83	52.89	0.78	14.97	0.05
MK	13.65	14.76	0.92	-6.97	4.97	-0.52	8.04	-0.08

Panel B: Predictor variables								
	Mean(%)	SD(%)	Min(%)	Max(%)	Skewness	Kurtosis	AR1	p_{df}
VIX	16.30	5.53	9.14	48.00	2.05	8.36	0.95	0.00
BIL	0.32	0.48	-0.02	1.91	1.92	5.61	1.00	1.00
TE	1.99	0.61	0.87	3.60	0.45	2.50	1.00	0.89
DE	0.95	0.25	0.53	1.54	0.62	2.33	1.00	0.31

Table 2: In-sample predictability of Bitcoin returns

This table presents estimates of predictive regressions of the form: $r_{t+1} = a + b'X_t + \epsilon_{t+1}$, where r_{t+1} denotes the return on Bitcoin on business day $t+1$. In Panel A, the predictors are the log price/moving average ratios, $pma_t(L)$, where L is the number of weeks, with 5 business days per week. We also extract the first three principal components ($PC1, PC2, or\ PC3$) from the $pma_t(L)$. In columns (1) to (5) of Panel B, the predictors include these principal components along with the other return predictors ($VIX, BILL, TERM,$ and DEF). Column (6) of Panel B adds the three principal components. The sample period are business days 12/06/2010–6/30/2018 ($n =$ 1,976). Heteroscedasticity-robust t-statistics are presented in parentheses.

Panel A: 1-Day Predictability of Bitcoin returns by log moving average/price ratios							
	(1)	(2)	(3)	(4)	(5)		(6)
$pma(1)$	0.39					$PC1$	-0.01
	(2.61)						(-1.46)
$pma(2)$		0.40				$PC2$	2.64
		(2.67)					(2.86)
$pma(4)$			0.42			$PC3$	-5.01
			(2.81)				(1.82)
$pma(10)$				0.46			
				(3.02)			
$pma(20)$					0.45		
					(3.00)		
Adj-R^2 (%)	0.41	0.45	0.51	0.62	0.64		1.88

Panel B: 1-Day Predictability of Bitcoin returns by log moving average/price ratios						
	(1)	(2)	(3)	(4)	(5)	(6)
VIX	-0.04				-0.05	-0.02
	(-1.33)				(-1.30)	(-0.34)
$BILL$		-0.11			0.05	0.02
		(-0.71)			(1.08)	(1.91)
$TERM$			0.34		0.34	-0.04
			(1.01)		(-0.49)	(-0.59)
DEF				-0.43	-0.75	-0.01
				(-1.68)	(-1.85)	(1.53)
$PC1$						-0.01
						(2.32)
$PC3$						2.24
						(2.50)
$PC3$						5.30
						(1.84)
Adj-R^2 (%)	0.01	0.00	0.00	0.00	0.20	1.82

Table 3: In-sample predictability of Bitcoin returns conditional on variance

This table presents estimates of predictive regressions of the form:

$$r_{t+1} = a + b \cdot pma_t(L) + c \cdot \sigma_t^2 + d \cdot pma_t(L) \cdot \sigma_t^2 + \epsilon_{t+1},$$

where r_{t+1} denotes the return on Bitcoin on day $t+1$, $pma_t(L)$ denotes the log price-to-L-week moving average ratio, and σ_t^2 denotes the exponential weighted moving average variance of Bitcoin returns. The σ_t^2 are defined recursively as $\sigma_t^2 = 0.94 \cdot \sigma_{t-1}^2 + 0.06 \cdot r_t^2$. The sample period for the regression is 12/06/2010−6/30/2018 (n=2,766 using 7-day-per week observations). The initial σ_0^2 is estimated as the sample variance of r_t over 7/28/2010–12/05/2010. Heteroskedasticity-robust t-statistics are presented in parentheses. *, **, *** denotes 10%, 5%, 1% significance levels.

	(1)	(2)	(3)	(4)	(5)
$pma(1)$					
	(2.13)				
$pma(1) \cdot \sigma^2$	-5.03				
	(-1.31)				
$pma(2)$		7.52***			
		(3.29)			
$pma(2) \cdot \sigma^2$		-5.57**			
		(-2.07)			
$pma(4)$			5.81***		
			(4.01)		
$pma(4) \cdot \sigma^2$			-3.95**		
			(-2.43)		
$pma(10)$				2.87***	
				(3.55)	
$pma(10) \cdot \sigma^2$				-1.83**	
				(-2.08)	
$pma(20)$					1.26***
					(2.59)
$pma(20) \cdot \sigma^2$					-0.98
					(-1.56)
σ^2	1.04**	1.28***	1.50***	1.51**	1.56**
	(2.12)	(2.70)	(2.97)	(2.51)	(2.24)
Adj-R^2 (%)	0.93	1.62	2.10	1.77	1.09

Table 4: Out-of-sample predictability of Bitcoin returns

Panels A and B present R^2_{OS} (out-of-sample R^2) in percent for recursively estimated predictive regressions of the form: $r_{t+1} = a + b'X_t + \epsilon_{t+1}$, where r_{t+1} denotes day-$t+1$ return on Bitcoin. Both panels use 5-day-per week observations. In Panel A, the predictors are the $pma(L)$ and in Panel B, they are $VIX, BILL, DEF,$ and $TERM$. Panel C uses the 7-day-per-week-observations and forecasts one week returns ($r_{t+1,t+7}$) using recursively estimated regressions of the form: $r_{t+1,t+7} = a + b'X_t + \epsilon_{t+1,t+7}$. T_0 denotes the in-sample period as a percentage of the total sample. The MEAN is a simple combination forecast that averages the five moving average forecasts. The sample is 12/06/2010−6/30/2018 (n=1976 in Panels A and B; n=2766 for Panel C). *, **, *** denotes 10%, 5%, 1% significance levels using the Clark-West (2007) MSFE-adjusted statistic that tests the null of equal MSFE (R^2_{OS}=0) against the competing model that has a lower MSFE (R^2_{OS}>0).

Panel A: 1-day horizon, 5-day-per-week observations						
T_0	$pma(1)$	$pma(2)$	$pma(4)$	$pma(10)$	$pma(20)$	MEAN
25%	-0.32	-0.11	0.70**	1.01**	0.72**	0.83*
50%	-0.73	-0.27	0.08	0.31**	0.86***	0.38*
90%	0.94	1.13*	1.51*	0.81	0.70	1.42*
Panel B: 1-day horizon, 5-day-per-week observations						
T_0	*VIX*	*BILL*	*TERM*	*DEF*		MEAN
25%	-0.87	-0.21	-0.27	-0.01		-0.14
50%	-1.07	0.06	-0.06	-0.03		-0.13
90%	-0.77	-0.08	-0.02	0.17		-0.10
Panel C: 1-week horizon, 7-day-per-week observations						
T_0	$pma(1)$	$pma(2)$	$pma(4)$	$pma(10)$	$pma(20)$	MEAN
25%	-0.11	0.84**	3.67**	3.71**	1.66**	3.08**
50%	1.05**	1.06**	2.38**	2.87**	4.13**	3.62**
90%	2.70**	2.32*	2.57**	1.07*	1.66**	1.92**

Table 5: Performance of Bitcoin trading strategies

This table presents summary statistics of the returns in excess of the 1-day risk-free rate on Bitcoin (BTC) and each of the MA(L) Bitcoin strategies, which take a long position in Bitcoin if $pma_t(L) > 0$, and the risk-free rate otherwise. EW denotes an equal-weighted portfolio of the individual $MA(L)$ strategies. Means, standard deviations, and Sharpe ratios are annualized. The sample period is daily from 12/06/2010−6/30/2018 (7 days per week). Panel A presents full sample results (n=2,764). Panels B and C, respectively, present results for the first (9/17/2010-8/28/2014, n=1,383) and second halves (8/29/2014-6/30/2018, n=1,383) of the sample. MDD denotes maximum drawdown. We use Ledoit and Wolf (2008) test of equality of Sharpe ratios that is robust to heteroskedasticity and serial correlation. *, **, *** denotes significance at the 10%, 5%, and 1% confidence levels, respectively.

Panel A: Full-sample								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
BTC	193.18	106.20	1.82	-38.83	52.89	0.78	14.97	89.48
MA(1)	196.54	79.33	2.48**	-38.83	52.89	2.12	31.47	71.65
MA(2)	187.38	79.20	2.37**	-38.83	52.89	2.07	31.27	64.43
MA(4)	187.34	82.67	2.27*	-38.83	52.89	1.63	29.21	69.66
MA(10)	195.70	88.50	2.21*	-38.83	52.89	1.59	24.99	70.28
MA(20)	188.77	94.96	1.99	-38.83	52.89	1.27	21.11	77.87
EW	191.15	78.72	2.43***	-38.83	52.89	2.09	31.31	64.60
Panel B: First-half								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
BTC	288.46	129.50	2.23	-38.83	52.89	0.77	12.47	89.48
MA(1)	279.04	98.77	2.83*	-38.83	52.89	1.85	24.58	71.65
MA(2)	272.33	99.01	2.75	-38.83	52.89	1.88	24.32	64.43
MA(4)	277.63	104.11	2.67	-38.83	52.89	1.45	22.16	69.66
MA(10)	309.57	110.78	2.79**	-38.83	52.89	1.48	19.20	70.28
MA(20)	271.54	118.56	2.29*	-38.83	52.89	1.17	16.26	77.87
EW	282.02	99.41	2.84**	-38.83	52.89	1.85	23.54	64.60
Panel C: Second-half								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
BTC	97.89	75.80	1.29	-21.90	25.41	0.10	8.14	69.77
MA(1)	114.03	52.90	2.16**	-11.13	25.41	1.68	16.37	32.82
MA(2)	102.43	52.02	1.97*	-11.13	22.97	1.02	12.81	37.27
MA(4)	97.05	52.81	1.84	-14.24	22.97	0.66	12.24	43.85
MA(10)	81.83	57.67	1.42	-16.73	22.97	0.13	11.34	56.33
MA(20)	106.01	62.82	1.69	-16.73	25.41	0.41	12.03	58.44
EW	100.27	49.70	2.02**	-11.13	22.97	1.00	12.10	40.05

Table 6: Alphas of MA Bitcoin strategies relative to buy-and-hold benchmark

Panels A and B present regressions of the form: $rx_t^{\mathrm{MA}(L)} = \alpha + \beta \cdot rx_t + \epsilon_t$, where rx_t denotes the day-t buy-and-hold excess return on Bitcoin and $rx_t^{\mathrm{MA}(L)}$ denotes the excess return on the $\mathrm{MA}(L)$ Bitcoin strategy. Beneath each regression is the appraisal ratio of the MA strategy as well as the utility gain from access to $rx_t^{\mathrm{MA}(L)}$ in addition to rx_t. EW denotes an equal-weighted portfolio of the MA strategies. Panel A also reports the average daily turnover (TO) of the MA strategies and the one-way transaction cost (FEE) that would be required to eliminate the alpha of the MA strategy. Panel A presents results for the full sample period (12/06/2010−6/30/2018, n=2,766). Panel B presents results for the second half of the sample (n=1,383). Heteroskedasticity-robust t-statistics are below point estimates in parentheses. *, **, *** denotes significance at the 10%, 5%, and 1% confidence levels, respectively.

Panel A: Full-sample						
	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
β	0.56***	0.56***	0.61***	0.69***	0.80***	0.64***
	(15.65)	(15.52)	(17.70)	(23.02)	(34.57)	(24.08)
α(%)	0.24***	0.22***	0.19***	0.17***	0.09**	0.18***
	(4.66)	(4.21)	(3.75)	(3.37)	(2.12)	(4.71)
R^2	0.56	0.56	0.61	0.69	0.80	0.75
Appraisal	1.68	1.52	1.35	1.26	0.81	1.71
Utility gain(%)	85.53	69.39	55.45	47.84	19.70	88.15
TO(%)	17.62	10.35	6.15	2.93	1.37	7.68
FEE(%)	1.38	2.12	3.13	5.75	6.85	2.39

Panel B: Second-half subsample						
	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
β	0.49***	0.47***	0.49***	0.58***	0.69***	0.54***
	(12.84)	(12.51)	(12.76)	(14.70)	(19.56)	(19.29)
α(%)	0.18***	0.15***	0.14**	0.07	0.11**	0.13***
	(3.41)	(2.90)	(2.54)	(1.31)	(2.10)	(3.27)
R^2	0.49	0.47	0.48	0.58	0.69	0.68
Appraisal	1.75	1.49	1.31	0.67	1.10	1.69
Utility gain(%)	183.90	132.81	102.43	27.17	72.90	170.74

Table 7: Performance of trading strategies applied to Ripple and ETH

Panel A presents summary statistics of the returns in excess of the 1-day risk-free rate on Ripple (XRP) and each of the MA(L) strategies applied to Ripple. Means, standard deviations, and Sharpe ratios are annualized. MDD denotes maximum drawdown. EW denotes an equal-weighted portfolio of the MA strategies. Panel B presents regressions of the form: $rx_t^{\mathrm{MA}(L)} = \alpha + \beta \cdot rx_t + \epsilon_t$, where rx_t denotes the day-t buy-and-hold excess return on XRP and $rx_t^{\mathrm{MA}(L)}$ denotes the excess return on the MA(L) XRP strategy. Beneath each regression is the appraisal ratio of the MA strategy and the utility gain from access to $rx_t^{\mathrm{MA}(L)}$. In Panels A and B, the sample is 12/24/2013–6/30/2018 (n=1,650). Panels C and D, presents similar statistics as Panels A and B, respectively, but for strategies applied to Ethereum (ETH) instead of XRP. In Panels C and D, the sample is 12/28/2015–6/30/2018 (n=916). We use the Ledoit and Wolf (2008) test of equality of Sharpe ratios. Heteroskedasticity robust t-statistics are below point estimates in parentheses. *, **, *** denotes significance at the 10%, 5%, and 1% confidence levels, respectively.

Panel A: Summary Statistics for XRP strategies

	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
XRP	167.15	159.61	1.05	-46.01	179.37	7.54	141.85	90.22
MA(1)	222.36	140.17	1.59**	-46.01	179.37	10.90	234.94	77.60
MA(2)	222.98	140.98	1.58**	-46.01	179.37	10.85	230.83	72.41
MA(4)	196.70	140.42	1.40	-46.01	179.37	10.93	234.77	57.93
MA(10)	164.11	141.39	1.16	-46.01	179.37	10.76	228.61	81.63
MA(20)	135.51	143.88	0.94	-46.01	179.37	10.24	213.68	85.50
EW	188.33	136.34	1.38*	-46.01	179.37	11.86	262.17	65.72

Panel B: Strategy alphas for XRP strategies

	(1)	(2)	(3)	(4)	(5)	(6)
	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
β	0.77***	0.78***	0.77***	0.78***	0.81***	0.79***
	(10.99)	(11.87)	(11.36)	(11.87)	(13.83)	(12.34)
α(%)	0.26***	0.25***	0.18**	0.09	-0.00	0.16**
	(3.24)	(3.20)	(2.33)	(1.15)	(-0.01)	(2.49)
R^2	0.77	0.78	0.78	0.78	0.81	0.84
Appraisal	1.40	1.41	1.01	0.50	0.00	1.06
Utility gain(%)	178.29	180.09	92.91	22.97	0.00	102.93

Table 7: (Cont'd)

Panel C: Summary Statistics for ETH strategies								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
ETH	334.16	132.00	2.53	-27.06	35.36	0.81	6.90	73.48
MA(1)	319.09	107.37	2.97	-27.06	35.36	1.68	12.49	42.36
MA(2)	310.48	108.48	2.86	-27.06	35.36	1.61	12.15	43.11
MA(4)	345.23	111.45	3.10*	-27.06	35.36	1.41	11.44	56.81
MA(10)	247.73	116.86	2.12	-27.06	35.36	1.10	10.13	75.23
MA(20)	302.12	122.65	2.46	-27.06	35.36	1.05	8.77	69.92
EW	304.93	107.28	2.84	-27.06	35.36	1.57	12.23	50.41

Panel D: Strategy alphas for ETH strategies						
	(1)	(2)	(3)	(4)	(5)	(6)
	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
β	0.66***	0.67***	0.71***	0.78***	0.86***	0.74***
	(17.83)	(18.49)	(21.39)	(26.36)	(39.89)	(27.89)
α(%)	0.27**	0.23**	0.29***	-0.04	0.04	0.16**
	(2.55)	(2.23)	(2.90)	(-0.41)	(0.50)	(2.17)
R^2	0.66	0.67	0.71	0.78	0.86	0.82
Appraisal	1.57	1.37	1.79	0.00	0.31	1.30
Utility gain(%)	38.39	29.39	50.07	0.00	1.48	26.25

Table 8: Performance of trading strategies applied to NASDAQ over 1996–2005

Panel A presents summary statistics of the returns in excess of the 1-day risk-free rate on NASDAQ and each of the MA(L) NASDAQ strategies. Means, standard deviations, and Sharpe ratios are annualized. EW denotes an equal-weighted portfolio of the MA strategies. MDD denotes maximum drawdown. Panel B presents regressions of the form: $rx_t^{\mathrm{MA}(L)} = \alpha + \beta \cdot rx_t + \epsilon_t$, where rx_t denotes the day-t buy-and-hold excess return on NASDAQ and $rx_t^{\mathrm{MA}(L)}$ denotes the excess return on the MA(L) NASDAQ strategy. Beneath each regression is the appraisal ratio of the MA strategy and the utility gain from access to $rx_t^{\mathrm{MA}(L)}$. The sample period is 1/2/1996−12/30/2005 (n=2,519). Panel C present results similar to Panel A using over the 1998–2002 subsample (n=1,256). We use the Ledoit and Wolf (2008) test of equality of Sharpe ratios. Heteroskedasticity-robust t-statistics are below point estimates in parentheses. *, **, *** denotes significance at the 10%, 5%, and 1% confidence levels, respectively.

Panel A: Summary Statistics of NASDAQ strategies

	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
NASDAQ	8.53	29.01	0.29	-9.69	14.15	0.19	7.14	77.93
MA(1)	9.32	18.34	0.51	-6.23	8.10	0.07	9.55	42.73
MA(2)	12.80	17.63	0.73*	-6.23	8.10	0.04	9.17	42.08
MA(4)	13.34	17.41	0.77*	-5.59	8.10	-0.07	8.36	25.66
MA(10)	13.84	17.45	0.79*	-7.66	4.92	-0.40	7.37	33.81
MA(20)	7.78	17.20	0.45	-7.66	4.28	-0.50	7.68	45.62
EW	11.42	15.09	0.76**	-5.58	4.86	-0.16	5.97	34.49

Panel B: Strategy alphas

	(1)	(2)	(3)	(4)	(5)	(6)
	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
β	0.40***	0.37***	0.36***	0.36***	0.35***	0.37***
	(16.66)	(16.23)	(16.28)	(16.53)	(16.32)	(19.97)
α(%)	0.02	0.04**	0.04**	0.04**	0.02	0.03**
	(1.32)	(2.18)	(2.33)	(2.44)	(1.09)	(2.47)
R^2	0.40	0.37	0.36	0.36	0.35	0.50
Appraisal	0.42	0.69	0.74	0.77	0.34	0.78
Utility gain(%)	199.85	548.18	627.66	687.82	137.42	105.47

Panel C: Summary Statistics of NASDAQ strategies over 1998–2002

	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
NASDAQ	-0.13	37.18	0.00	-9.69	14.15	0.23	5.08	77.93
MA(1)	9.62	23.07	0.42	-6.23	8.10	0.06	7.17	42.73
MA(2)	10.69	21.93	0.49*	-6.23	8.10	0.05	7.14	42.08
MA(4)	12.84	21.47	0.60*	-5.59	8.10	-0.07	6.62	25.66
MA(10)	11.89	21.12	0.56*	-7.66	4.92	-0.43	6.10	33.81
MA(20)	7.11	19.98	0.36	-7.66	4.28	-0.48	6.59	39.75
EW	10.43	18.23	0.57**	-5.58	4.86	-0.15	4.83	34.49

Table 9: Predictability of returns and performance of MA strategies across size portfolios

In Panels A and B, we apply our MA strategies to each of the three value-weighted Fama and French (1993) size portfolios, "Small", "Medium", and "Big". Panel A presents heteroskedasticity-robust t-statistics from regressions of daily excess portfolio returns on the L-week price-to-moving average ratios:

$$rx_{t+1} = a + b \cdot pma_t(L) + \epsilon_{t+1}.$$

Panel B presents Sharpe ratios for the buy-and-hold (BH) return on each portfolio as well as each of the MA strategies and the equal-weighted portfolio (EW) of the MA strategies. Panels C and D repeat the analysis of Panels A and B, respectively, using age-sorted tercile portfolios, "Young", "Medium", and "Old". The sample period is July 1, 1963 through June 30, 2018.

Panel A: t-statistics from predictive regressions for returns of size portfolios

	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)
Small	6.03	5.90	5.87	5.23	3.96
Medium	4.43	3.42	3.06	2.44	1.67
Big	-0.10	-0.92	-0.96	-0.83	-0.67

Panel B: Sharpe ratios of MA strategies applied to size portfolios

	BH	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
Small	0.50	2.05	1.90	1.75	1.52	1.16	1.91
Medium	0.51	1.73	1.52	1.27	1.10	0.87	1.49
Large	0.39	0.73	0.62	0.53	0.48	0.47	0.65

Panel C: t-statistics from predictive regressions for returns of age portfolios

	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)
Young	6.56	4.97	4.49	3.70	2.89
Medium	3.96	2.59	2.14	1.82	1.33
Old	0.62	-0.25	-0.33	-0.34	-0.32

Panel D: Sharpe ratios of MA strategies applied to age portfolios

	BH	MA(1)	MA(2)	MA(4)	MA(10)	MA(20)	EW
Young	0.48	2.00	1.75	1.57	1.35	1.12	1.79
Medium	0.46	1.53	1.22	1.16	0.95	0.85	1.31
Old	0.47	0.91	0.73	0.61	0.59	0.63	0.80

Table 10: Predictability of returns and performance of MA strategies across portfolios formed on size and analyst coverage

At the end of each year, we form nine value-weighted portfolios formed as the intersections of two independent tercile sorts of all U.S. common stocks into three portfolios based on each of market capitalization ("Small", "Medium", and "Big") and the number of analyst forecasts over the year ("Low", "Medium", and "High"). The first three columns of the table present heteroskedasticity-robust t-statistics from regressions of daily excess portfolio returns on the L-week price-to-moving average ratios:

$$rx_{t+1} = a + b \cdot pma_t(L) + \epsilon_{t+1}.$$

The fourth column of the table presents heteroskedasticity-robust GMM-based t-statistics of the difference between the b from to the "Low" portfolio in the row minus the b from the "High" portfolio in the row. The sample period is January 2, 1985 through June 29, 2018.

	Low	Med	High	Low-High
MA(1)				
Small	5.24	5.04	4.93	2.95
Med	1.48	1.83	2.72	-1.75
Big	0.19	0.00	-1.94	4.08
MA(2)				
Small	5.18	4.87	4.39	2.95
Med	1.16	1.34	1.94	-1.17
Big	-0.45	-0.63	-2.16	3.58
MA(4)				
Small	5.19	4.71	4.19	2.78
Med	1.18	1.33	1.68	-0.66
Big	-0.45	-0.64	-1.83	2.97
MA(10)				
Small	4.98	4.40	3.32	2.80
Med	1.01	1.16	1.44	-0.47
Big	-0.08	-0.39	-1.39	2.76
MA(20)				
Small	3.94	3.10	2.29	2.35
Med	0.43	0.47	0.71	-0.31
Big	0.10	-0.43	-1.14	2.56

Table 11: Volume and technical trading indicators

This table presents regressions of the form:

$$\Delta \log(volume)_t = a + b \cdot X_t + c \cdot |r_t| + \epsilon_t,$$

$$\Delta \log(volume)_t = a + b \cdot X_t + c \cdot |r_t| + d \cdot \Delta \log(volume)_{t-1} + \epsilon_t,$$

where $volume_t$ denotes the trading volume in Bitcoin on day t, $|r_t|$ denotes the absolute return on Bitcoin on day t, and X_t denotes one of two predictors. The second equation introduces lagged volume to accommodate for possible serial correlation. In column (1), X_t is the sum $\left(\sum_L |\Delta S_{L,t}|\right)$ of the absolute turnovers $|\Delta S_{L,t}|$ from each of the MA(L) strategies. In column (2), X_t is the cross-sectional standard deviation $\left(\sigma_L(\Delta S_{L,t})\right)$ of $\Delta S_{L,t}$, a measure of the "disagreement" among technical traders using the different MA strategies ($L = 1, 2, 4, 10$, or 20 weeks). In column (3), X_t includes $\sum_L |\Delta S_{L,t}|$ and $\sigma_L(\Delta S_{L,t})$. The sample is 12/27/2013–6/30/2018 (n=1,647). Heteroskedasticity-robust t-statistics are in parentheses.

Panel A: Determinants of Volume, without controlling for lagged volume

	(1)	(2)	(3)
$\sum_L(\|\Delta S_{L,t}\|)$	0.03		0.07
	(4.38)		(1.47)
$\sigma_L(\Delta S_{L,t})$		0.15	0.05
		(3.65)	(3.46)
$\|r_t\|$	4.90	5.03	4.73
	(11.94)	(11.76)	(11.58)
Adj-R^2	0.17	0.16	0.17

Panel B: Determinants of Volume, controlling for lagged Volume

	(1)	(2)	(3)
$\sum_L(\|\Delta S_{L,t}\|)$	0.03		0.04
	(3.56)		(3.85)
$\sigma_L(\Delta S_{L,t})$		0.12	0.16
		(2.78)	(1.80)
$\|r_t\|$	5.07	5.18	5.04
	(13.05)	(12.91)	(12.69)
$\Delta \log(volume)_{t-1}$	-0.23	-0.23	-0.23
	(10.13)	(-10.21)	(10.81)
Adj-R^2	0.22	0.22	0.22

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